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MT/ D-13

BASICS OF STATE VARIABLE TECHNIQUE

Paper –MTEC–1.3

Time allowed : 3 hours] *[Maximum marks : 60*

Note : Attempt any five questions, selecting at least one question from each section. All questions carry equal marks. Assume missing data if any.

Section–I

1. (a) Show that a triangular orthogonal matrix is diagonal. Show that if the k-th diagonal entry of an upper triangular matrix is zero, then the first k columns of the matrix are dependent. 7

   (b) Use the SVD to establish the fundamental identity rank (A) = rank (A^T). 5

2. Prove that similar matrices have the same characteristic polynomial and therefore the same eigenvalues. Find rank and nullity of the following matrices. Also find the bases of the range space and the null space of the matrices A and B 6+6

$$A = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 6 & 8 & 3 & 4 \\ 9 & 12 & 4 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 9 \\ 4 & 1 & 18 \\ -6 & -3 & -27 \end{bmatrix}$$

Section–II

3. For the state equation $\dot{x}(t) = A(x)t$ with $A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix}$ Find the initial condition vector x(0) which will only excite the mode corresponding to the eigenvalue with the most negative real part. 12

4. Prove that the controllability and observability of a linear time invariant system are invariant under any equivalence transformation. 12

Section–III

5. For the system $\dot{x}(t) = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} x(t)$ find a suitable Lyapunov function $V(\dot{x})$. Obtain an upper bound on the response time such that it takes the system go from a point on the boundary of the closed curve $V(x) = 100$ to a point within the closed curve $V(x) = 0.05$. 12

6. Explain the methods used to find Lyapunov function for nonlinear systems. 12

Section–IV

7. For autonomous system $x(k+1) = fx(k)$; $f(0) = 0$ give the sufficient conditions for stability. 12

8. Give the Jury's stability algorithm and using this determine the stability of given system $4z^3 - 12z^2 + 13z - 7 = 0$. 12

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